Planelets: A New Analysis Tool for Planar Feature Extraction

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Overview

Locally planar structures, formed by sweeping edges of objects, are commonly found in video sequences and convey most of the useful information. In this paper, the issue of efficient representation of such structures is addressed. We propose a novel representation tool which uses basis functions, termed as *planelets*, resembling planar structures and having compact support in space-time and spatiotemporal frequency. The representation is translation invariant, offers good directional selectivity, and can be computed efficiently. We show that the new representation, while being fast, produces video denoising results which compare favourably to one of the best known methods.

Motivation

The performance of wavelet based algorithms for processing and analysis of video sequences is severely restricted due to the following observation. While the wavelet transform in higher dimensions can be conveniently computed separably, separability also seriously limits the ability of wavelets to efficiently represent higher dimensional features (such as lines in images or planes in 3D image volumes). Furthermore, the lack of frequency selectivity remains an elusive problem with most techniques operating in the wavelet domain.

Planar Features : Locally planar structures, such as moving luminance edges, are commonly found in video sequences and often convey most of the information. In this paper, a novel representation designed specifically for efficiently representing 3D functions with planar singularities is presented.

Applications : Extraction of such planar features may be useful in various applications, such as video denoising, video coding, geometry estimation and tracking of objects in video sequences.

Representation with Planelets

Continuous Planelet Transform : The planelet basis has a combination of scale, translation, and directional characteristics which are well matched to the locally planar surfaces of interest in many applications. A prototypical planelet function in 1D is of the following form

$$f_{\xi,\omega,a}(x) = g\left(\frac{x-\xi}{a}\right)\exp\left[-j\frac{\omega(x-\xi)}{a}\right] \quad (1)$$

where ξ,ω and a are respectively the location, frequency and scale parameters of the function. The function $g(.)$ is a window function, chosen alongwith the sampling interval to ensure invertibility of the discrete form of the transform. In 2D, the planelet basis can be regarded as a modification of the complex wavelet bases proposed by Kingsbury, which show both translation invariance and directional selectivity, and may be used as an alternative to the ridgelet representation. In 3D, the basis comprises of the set of Cartesian products over ξ,ω at each scale a.

Discrete Planelet Transform : The discrete planelet transform (DPT) is a combination of two well known image transforms: the Laplacian pyramid and the windowed Fourier transform (WFT). The DPT of a video sequence x, in vector form, at scale m is given by

$$\hat{X}_m = \mathcal{F}_n(I - G_{m,m+1}G_{m+1,m})x_m \quad (2)$$

where $\hat{X}_m$ denotes the DPT at scale m, $\mathcal{F}_n$ is the WFT operator with window size $n\times n\times n$, I is the identity operator, x_m is the Gaussian pyramid representation of x at level m

$$x_m = \prod_{0}^{m-1} G_{l+1,l}\,x \quad (3)$$

and $G_{m,m+1}, G_{m+1,m}$ are the raising and lowering operators associated with transitions between levels in the Gaussian pyramid. Invertibility follows directly from equations (2) and (3):

Theorem 1 *The representation defined by equation (2) is invertible.*

Features :

- The planelet representation offers translation invariance, good directional selectivity.
- The computational complexity of DPT is $O(n)$, where n is the number of points in analysis window.
- In its current form, the representation provides a non-orthogonal basis and is redundant by less than 14%.

Basis Functions : The planelet basis functions have compact support in both space-time and spatiotemporal frequency. As shown in the Figure below, they resemble planar wave functions.

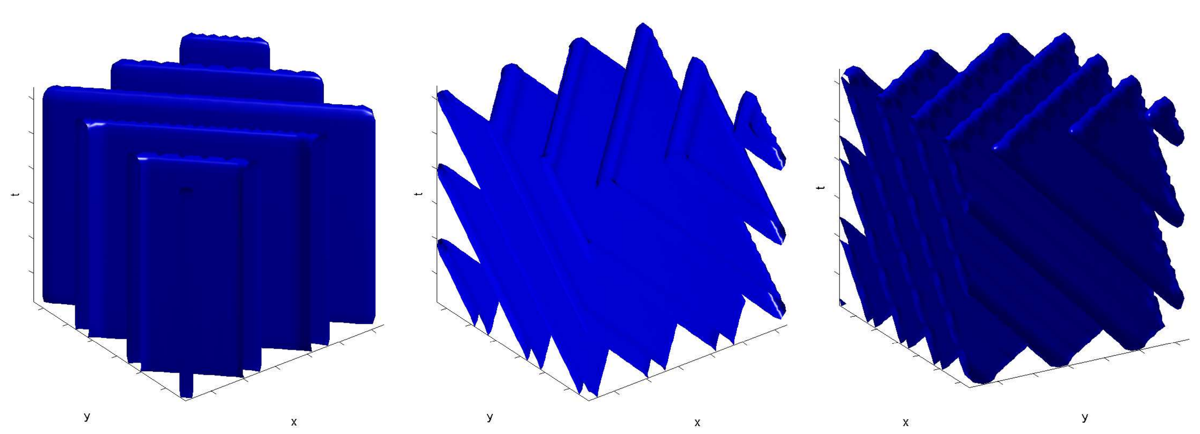

FIGURE 1. *Planelet basis functions*

Extracting Features with Planelets

The Idea : Planelets provide an ideal tool for representing local planes in a video sequence (or an image volume, in general) due to their ability to localise planar surfaces which correspond to lines in the Fourier domain.

How ? The presence of planar surface in a local analysis window can be inferred by computing the eigenvalues of the local inertia tensor in the window and analysing them. The parameters for orientation of the local planar surface and translation from centre of the window can also be estimated by analysing the most significant coefficients in the locality.

Example : Consider a video sequence synthesised by moving the centre of a circle on a sinusoidal wave in the time direction. Nonlinear approximations of this sequence using only 0.07% of the wavelet and planelet coefficients are shown in the Figure below. It is clear from this example that the planelet approximation of a video sequence containing locally planar surfaces can result in a smaller approximation error as compared to that using wavelets. Planelets, therefore, can also be used for a piecewise planar approximation of a video sequence.

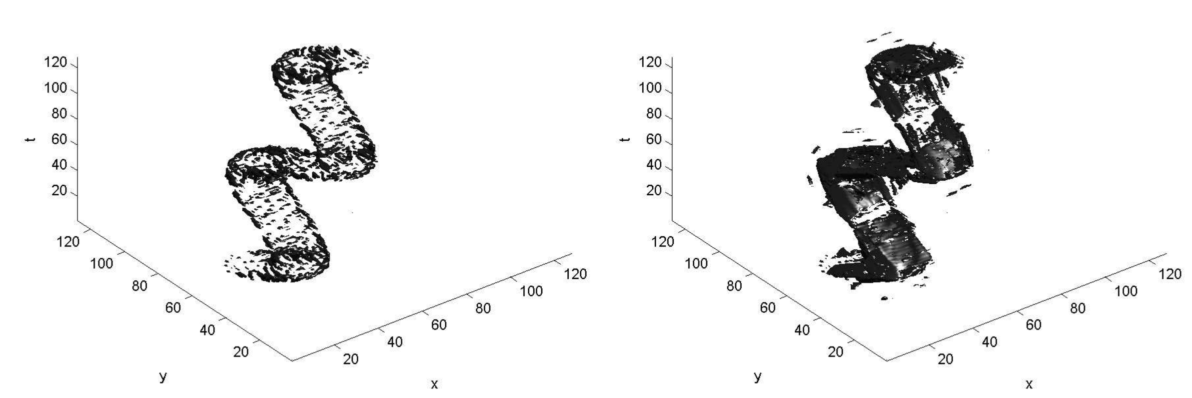

FIGURE 2. *Nonlinear approximation of a video sequence using wavelets (*Left*) and planelets (*Right*)*

Video Application

Application : In order to demonstrate the ability of planelets to capture locally planar structures, we conducted experiments with their application to **video denoising**.

Strategy : A simple soft thresholding in the planelet domain can be used for video denoising. A translation invariant (TI) version of the planelet denoising was implemented for a fair comparison with the translation invariant wavelet (TIW) denoising [**?**] in its 3D form.

Results : The algorithms described above for video denoising were tested on four standard video sequences reduced to a resolution of 128^3: *Miss America*, *Football*, *Hall*, and *Tennis*. The image data was corrupted with additive Gaussian noise, and adaptive thresholding was applied to the transform coefficients of the noisy sequence represented in a 3-level planelet domain using a 16^3 window. Denoising using TI planelet representation with *SURE* threshold generally outperforms the other two methods. Selected frames, for each of these sequences, restored by planelet denoising are shown in the Figure below. Two types of artifacts were observed from non-TI results: blocking artifacts due to the use of a 16^3 window, and *fake textures* which sometimes persist within these windows, due to suppression of a significant amount of high frequency energy. The TI planelet denoising proves to be effective in removing both these kind of artifacts, and is particularly good in reconstructing some of the details that were smoothed out with the TIW method.

Brief Analysis : The **computational complexity** of our algorithm is $O(n)$ as compared to $O\left(N + l^3 N\right)$ for TIW denoising, where n and N respectively denote the size of analysis window and the size of video sequence (resolution of each frame times the number of frames) and l is length of the wavelet filter. While being faster by orders of magnitude, our algorithm still compares favourably to the 3D TIW for all our experiments.

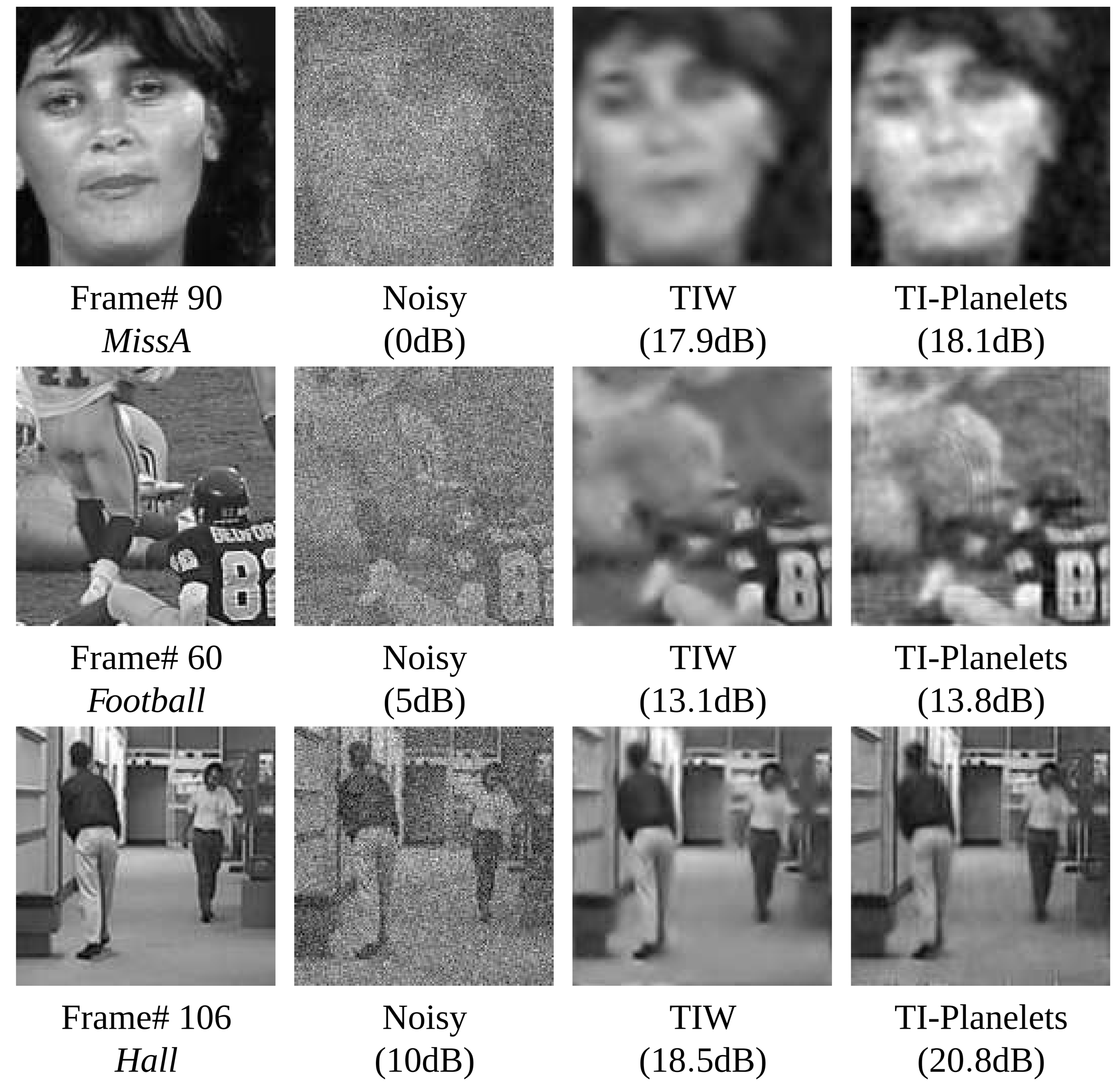

FIGURE 3. *Results of Video Denoising*

Summary

Conclusions :

- In this paper, planelets were proposed as an efficient representation tool for 3D functions with planar singularities, which are commonly found in video sequences in the form of moving luminance edges.
- It was shown that a piecewise planar approximation of a video sequence can be obtained by using a very small fraction of transform coefficients in the planelet domain.
- The ability of planelets to extract planar features from a video sequence makes them an attractive tool for analysis in various applications.

Future Directions :

- Although the discrete planelet transform can be computed efficiently, its redundancy and high storage requirements may be of concern in some applications. Future work will address these issues.
- An investigation into the usefulness of planelets in a wide range of video analysis applications.

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