Lecture 5

Tuesday, August 25, 2015
7:44 AM

- Las Vegas, Monte Carlo
- Random variables, expectations, linearity of expectations
- Expected running time
- Recursive majority
- Karger's algorithm
- Indicator variables
- Birthday paradox
- Coupon collector
- Randomized selection

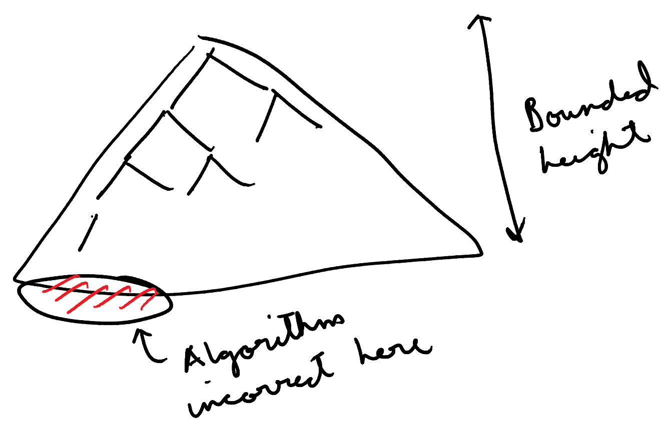
Randomized algorithms

Usual algorithms with access to a stream of perfectly random bits (0 or 1 with equal probability)

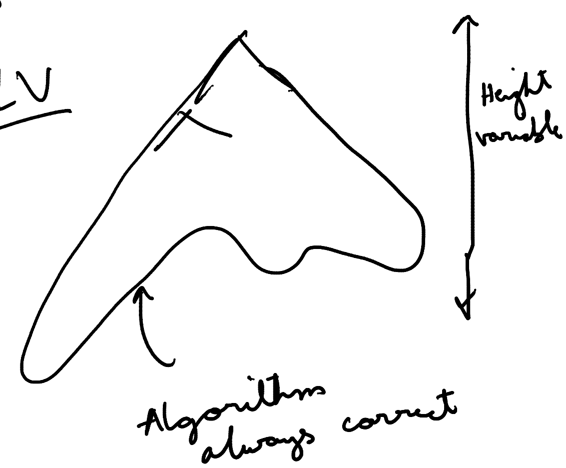
Two types of algorithms:

- Monte Carlo: Algorithm terminates in finite time but may output the wrong answer
- Las Vegas: Algorithm always outputs the correct answer but termination time is variable.

MC



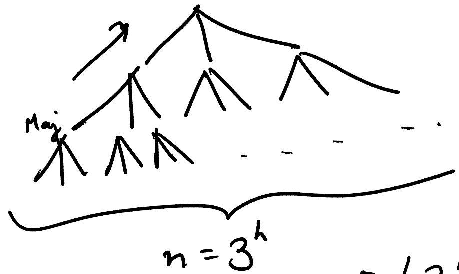
LV



Recursive majority

- Given Ternary tree of height h and $n = 3^h$ leaves, compute value at root
labels at leaves, ... (defined

given by majority of the values of its 3 children ...
recursively)



- Naive deterministic algorithm takes $O(3^h) = O(n)$ time. With randomness can do better!

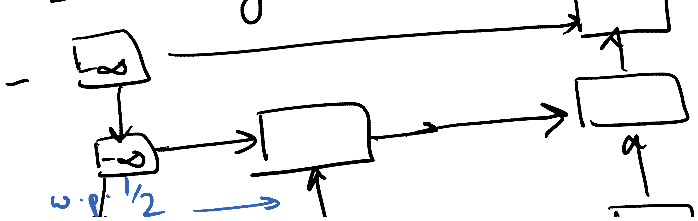
- def RANDOM-RM (v):

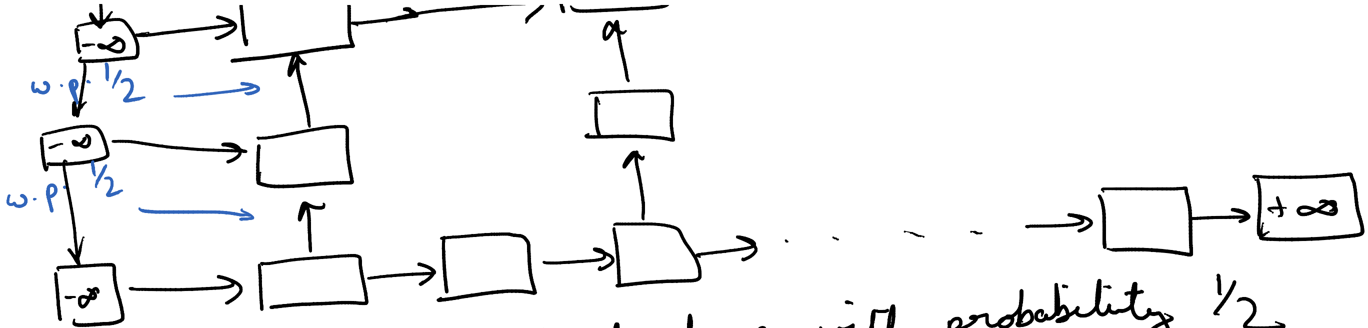
```
    if v is a leaf:
        return label(v)
    else:
        i,j <- two random elements of {1,2,3}
        x = RANDOM-RM(v.child(i))
        y = RANDOM-RM(v.child(j))
        if x == y:
            return x
        else:
            return RANDOM-RM(v.child(6-i-j))
```

- Let R_h be <u>expected</u> # of leaves examined if root is at height h
- <u>Claim</u>: $R_h \le \frac{8}{3} R_{h-1}$. (Proved in class)
- In expectation, only $\le \left(\frac{8}{3}\right)^h = n^{\log_3(8/3)}$ leaves examined!

Skip Lists

- Dictionary data structure with a spoonful of randomness
-





Each item copied to level above with probability 1/2

- Lemma : $\mathbb{E}[\text{height}] = O(\log n)$
- Pf : For a given item i, let h_i be height at i.
  Let $H = \max_i h_i$.

$$\Pr[h_i \geq k] = \frac{1}{2^k}$$

$$\Pr[H \geq k] = \Pr[\exists i \text{ s.t. } h_i \geq k]$$
$$\leq \frac{n}{2^k}.$$

$$\therefore \mathbb{E}[H] = \sum_{k=0}^{\infty} k \cdot \Pr[H = k]$$
$$= \sum_{k=0}^{\infty} \Pr[H > k]$$
$$\leq \sum_{k=0}^{2\log n} 1 + \sum_{k=2\log n+1}^{\infty} \frac{n}{2^k} = O(\log n).$$