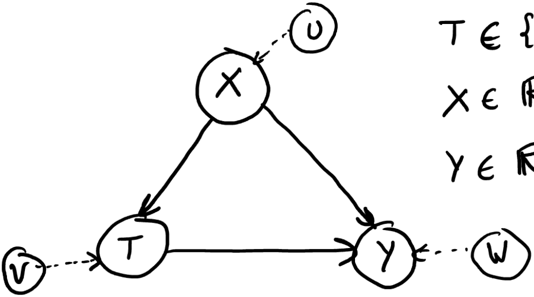
# Individual Treatment Effect Estimation & Causal Forests

Friday, June 19, 2020 9:48 PM



$T \in \{0, 1\}$

$X \in \mathbb{R}^d$ ✓

$Y \in \mathbb{R}$

X = Profile

T = treatment

Y = outcome

X = economy

T = pandemic

Y = recession?

$\Pr[X]$

$\Pr[T \mid x]$

$\Pr[Y \mid t, x]$

$$\boxed{\begin{aligned} X &= g(U) \\ T &= \cancel{g'(X) + V'} \quad 1 \\ Y &= g''(\underset{1}{\cancel{T}}, X) + W' \end{aligned}}$$

**Estimand:**

$$\underline{\underline{\text{I}}}\text{TE}(x) = \mathbb{E}[Y \mid do(T=1), X=x] - \mathbb{E}[Y \mid do(T=0), X=x]$$

$$= \mathbb{E}[Y \mid T=1, X=x] - \mathbb{E}[Y \mid T=0, X=x]$$

$$\boxed{= \mathbb{E}[g''(1,X) - g''(0,X)]}$$

**Assumption**: "Unconfoundedness"

**Assumption** (Overlap):

$$\forall x, \quad \Pr[T=1 \mid X=x] \in [\varepsilon, 1-\varepsilon].$$

**Problem**: Given samples

$(X_1, T_1, Y_1), \dots, (X_n, T_n, Y_n)$,

$\underline{\underline{\text{estimate}}}$ ITE $(x)$.

Want: $\mathbb{E}_x\left[(\widehat{\text{ITE}}(x) - \text{ITE}(x))^2\right] \leq \varepsilon.$

## Why not regression?

$$\begin{aligned}\mathrm{ITE}(x) &= \mathbb{E}[Y \mid T=1, X=x] \quad \leftarrow \\ &\quad - \mathbb{E}[Y \mid T=0, X=x] \quad \leftarrow\end{aligned}$$

Regularization bias.

## Theorem [Shalit - Johannsen - Sontag '17]

Suppose $h(x,t)$ is your estimate of $\mathbb{E}[Y \mid T=t, X=x]$.

$$\widehat{\mathrm{ITF}}(x) = h(x,1) - h(x,0).$$

**Then:**

$$\begin{aligned}&\mathbb{E}\left[\left(\widehat{\mathrm{ITE}}(x) - \mathrm{ITE}(x)\right)^2\right] \\ \lesssim\ &\mathop{\mathbb{E}}_{x}\left[\mathop{\mathbb{E}}_{y}\left[(h(x,1)-y)^2 \mid X=x, T=1\right] \,\middle|\, T=1\right] \\ &+ \mathop{\mathbb{E}}_{x}\left[\mathop{\mathbb{E}}_{y}\left[(h(x,0)-y)^2 \mid X=x, T=0\right] \,\middle|\, T=0\right] \\ &+ \mathrm{dist}\left(X \mid T=1,\ X \mid T=0\right)\end{aligned}$$

$\uparrow$ IPM distance based on hyp class containing $h$.

---

## Possible Approaches

① Apply "domain adaptation" technique

② Apply regularization together instead of separately.

---

X-Learner [Samples $(X_1, T_1, Y_1), \dots$]

① $\hat{m}_0(x)$ estimate of $\mathbb{E}[Y \mid T=0, X=x]$

$\hat{m}_1(x)$ $\qquad \mathbb{E}[Y \mid T=1, X=x]$

② If $T_i = 1$,

$$\tilde{D}_i^1 = Y_i - \hat{m}_0(X_i) <$$

If $T_i = 0$, $\tilde{D}_i^0 = \hat{m}_1(X_i) - Y_i$

$\widehat{ITE}_1(x)$ estimate of $\overset{\swarrow 1}{ITE(x)}$

$(X_i, \underline{\tilde{D}_i^1})$ for all $i$ s.t. $T_i = 1$.

" $T_i = 0$.

$\widehat{ITE}_0(\bar{x})$

③ $ITE(x) = e(x) \cdot \widehat{ITE}_0(x) +$
$(1 - e(x)) \cdot \widehat{ITE}_1(x)$

where $e(x) \approx \Pr[T = 1 \mid X = x]$.

Conjecture: If $(0, X, Y)$, $(1, X, Y)$

are "nice" and

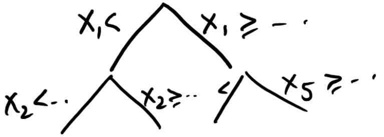
if $(0, X, m_1(X) - Y)$

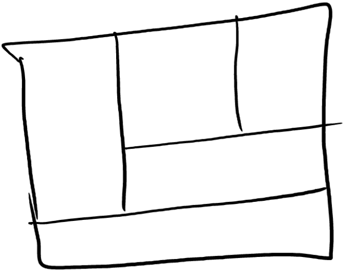
and $(1, X, Y - m_0(X))$

are also "nice",

then the X-learner converges fast.

# Causal Forests





$(X_1, Y_1), \ldots$

Estimate $\mathbb{E}[Y \mid X = x] = m(x)$

For forests, you randomize in 2 ways:

① Subsample from the training set

② Restrict to a random subset of variables at each step.

$$\hat{m}(x) = \frac{1}{|\{i : X_i \in L(x)\}|} \sum_{i : X_i \in L(x)} Y_i .$$

## Future Directions

① Understand "regularization bias" more precisely.

② Non-asymptotic / finite sample bounds

③ More complicated DAG's?
What if there are unmeasured confound~~ing~~ers? Robust statistics?

④ . . .