$$\underbrace{\underbrace{(t = x ; x = y)}_{\text{Swap}_1} ; y = t}_{\text{Swap}}$$

$$\vdash_{AR} x = x_0 \wedge y = y_0 \Rightarrow y = y_0 \wedge x = x_0 \quad \{ y = y_0 \wedge x = x_0 \}\ t = x\ \{ y = y_0 \wedge t = x_0 \}$$ Implied

$$\{ x = x_0 \wedge y = y_0 \}\ t = x\ \{ y = y_0 \wedge t = x_0 \}$$ Ass.

$$\{ y = y_0 \wedge t = x_0 \}\ x = y\ \{ x = y_0 \wedge t = x_0 \}$$ Ass.

$$\{ x = x_0 \wedge y = y_0 \}\ \text{Swap}_1\ \{ x = y_0 \wedge t = x_0 \}$$ Comp.

$$\{ x = y_0 \wedge t = x_0 \}\ y = t\ \{ x = y_0 \wedge y = x_0 \}$$ Ass.

$$\{ x = x_0 \wedge y = y_0 \}\ \text{Swap}\ \{ x = y_0 \wedge y = x_0 \}$$ Comp.

4.3.5. (b)

$\{\top\}$

$\{x + x + x = 3 \cdot x\}$ Implied

$y = x;$

$\{x + x + y = 3 \cdot x\}$ Assignment

$y = x + x + y$

$\{y = 3 \cdot x\}$ Assignment

$\vdash_{par} \{\top\}\ y = x;\ y = x + x + y\ \{y = 3 \cdot x\}$

$\vdash_{tot}$ -"-

4.3.12.

$\{ T \}$

$\{ x+1 = x+1 \}$ Implied

```
a = x+1;
```
$\{ a = x+1 \}$ Assignment

```
if (a-1 == 0) {
```
$\{ a = x+1 \wedge a-1 = 0 \}$ If-statement

$\{ 1 = x+1 \}$ Implied

```
    y = 1;
```
$\{ y = x+1 \}$ Assignment

```
} else {
```
$\{ a = x+1 \wedge \neg a-1 = 0 \}$ If-statement

$\{ a = x+1 \}$ Implied

```
    y = a;
```
$\{ y = x+1 \}$ Assignment

```
}
```

$\{ y = x+1 \}$ If-statement

Alternative If-statement rule

$$\frac{\{\phi_1\}\ C_1\ \{\psi\} \qquad \{\phi_2\}\ C_2\ \{\psi\}}{\{(B \to \phi_1) \wedge (\neg B \to \phi_2)\}\ \text{if } B \{C_1\} \text{ else } \{C_2\}\ \{\psi\}}$$

$\{ T \}$

$\{ (x+1 = 1 \rightarrow x = 0) \wedge (x+1 \neq 1 \rightarrow x+1 = x+1) \}$ Implied

a = x + 1 ;

$\{ (a = 1 \rightarrow x = 0) \wedge (a \neq 1 \rightarrow a = x+1) \}$ Assignment

$\{ (a-1 = 0 \rightarrow 1 = x+1) \wedge (\neg a-1 = 0 \rightarrow a = x+1) \}$ Implied

if (a - 1 == 0) {

$\{ 1 = x+1 \}$ If-statement

y = 1 ;

$\{ y = x+1 \}$ Assignment

} else {

$\{ a = x+1 \}$ If-statement

y = a ;

$\{ y = x+1 \}$ Assignment

}

$\{ y = x+1 \}$ If-statement

4.3.9. (c)

$$\dfrac{\dfrac{\vdash_{AR} \phi_1 \leftarrow \chi \wedge B \quad \{\phi_1\}\, C_1\, \{\psi\}}{\{\chi \wedge B\}\, C_1\, \{\psi\}}\ \text{Implied} \qquad \dfrac{\vdash_{AR} \phi_2 \leftarrow \chi \wedge \neg B \quad \{\phi_2\}\, C_2\, \{\psi\}}{\{\chi \wedge \neg B\}\, C_2\, \{\psi\}}\ \text{Implied}}{\{\underbrace{(B \to \phi_1) \wedge (\neg B \to \phi_2)}_{\chi}\}\ \text{if } B \{C_1\} \text{ else } \{C_2\}\ \{\psi\}}$$

If-statement (original)

4.3.15.

$\{ y = y_0 \wedge y \geq 0 \}$

$\{ 0 = x \cdot (y_0 - y) \}$ Implied

$z = 0;$

$\{ z = x \cdot (y_0 - y) \}$ Assigment

while $(y \,!= 0)$ {

$\{ z = x \cdot (y_0 - y) \wedge y \neq 0 \}$ Invariant Hyp. $\wedge$ guard

$\{ z + x = x \cdot (y_0 - (y-1)) \}$ Implied

$z = z + x;$

$\{ z = x \cdot (y_0 - (y-1)) \}$ Assignment

$y = y - 1;$

$\{ z = x \cdot (y_0 - y) \}$ Assignment

}

$\{ z = x \cdot (y_0 - y) \wedge y = 0 \}$ Partial - while

$\{ z = x \cdot y_0 \}$ Implied

4.4.1. (c)

$\{ y = y_0 \wedge y \geq 0 \}$

$\{ 0 = x \cdot (y_0 - y) \wedge 0 \leq y \}$ Implied

$z = 0;$

$\{ z = x \cdot (y_0 - y) \wedge 0 \leq y \}$ Assigment

while $(y != 0)$ {

$\{ z = x \cdot (y_0 - y) \wedge y \neq 0 \wedge 0 \leq y = E_0 \}$ Inv. Hyp. $\wedge$ guard

$\{ z + x = x \cdot (y_0 - (y-1)) \wedge 0 \leq y - 1 < E_0 \}$ Implied

$z = z + x;$

$\{ z = x \cdot (y_0 - (y-1)) \wedge 0 \leq y - 1 < E_0 \}$ Assignment

$y = y - 1;$

$\{ z = x \cdot (y_0 - y) \wedge 0 \leq y < E_0 \}$ Assignment

}

$\{ z = x \cdot (y_0 - y) \wedge y = 0 \}$ Partial - while

$\{ z = x \cdot y_0 \}$ Implied