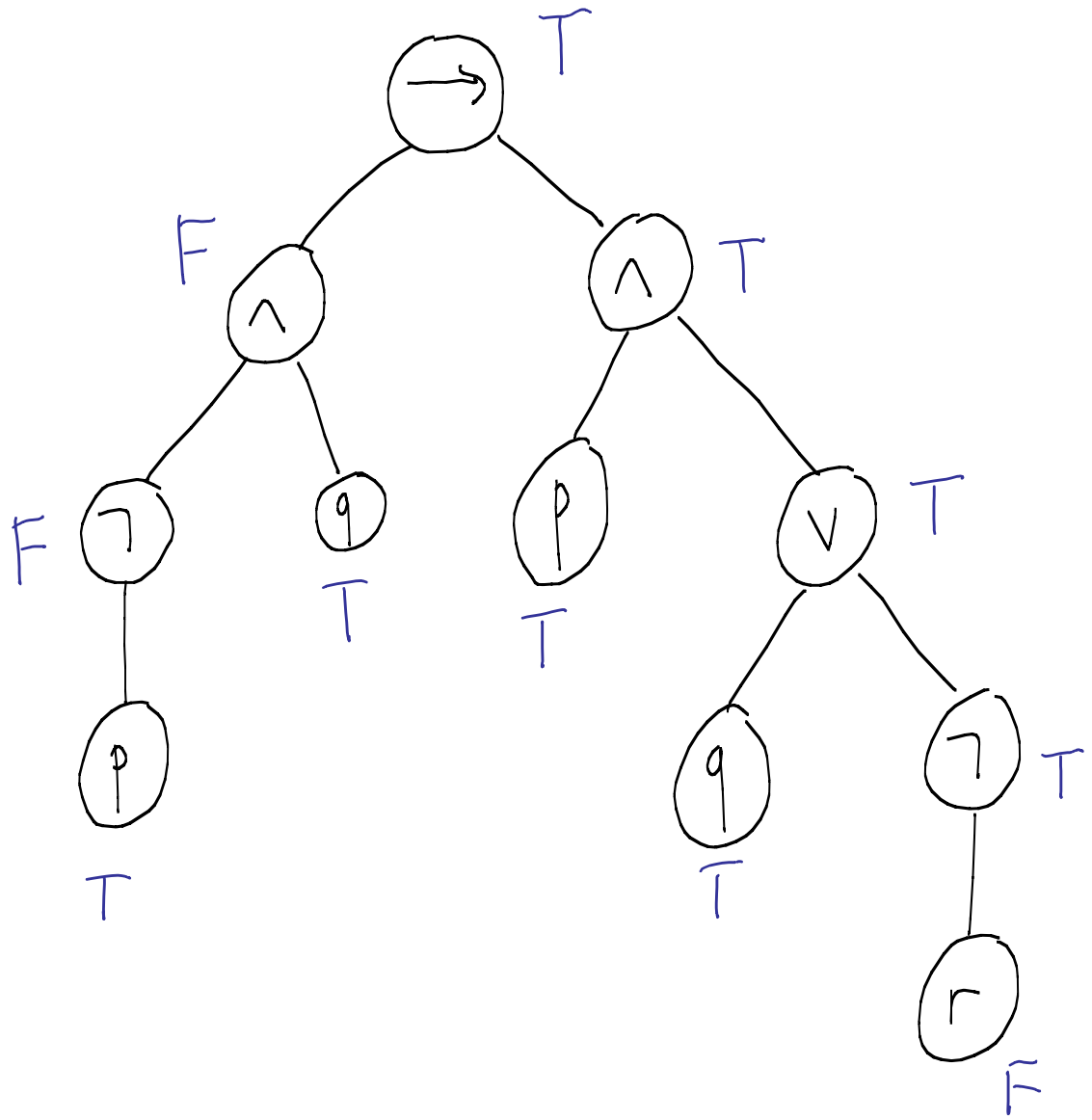
$$((\neg p) \wedge q) \rightarrow (p \wedge (q \vee (\neg r)))$$



p	q	$(p \rightarrow \neg q) \rightarrow (q \vee \neg p)$
F	F	T T T T T
F	T	T F T T T
T	F	T T F F F
T	T	F F T T F

p	q	$p \wedge q$	$\models$	p
F	F	F		F
F	T	F		F
T	F	F		T
T	T	T		T

valid semantic entailment ←

1	$p \wedge q$	premise
2	p	$\wedge e_1$ 1

$p \wedge q \vdash p$

Alternatively, $p \wedge q \models p$ is valid by the Soundness Th.

p	q	$p \to q$	$\models$	q
F	F	T		F
F	T	T		T
T	F	F		F
T	T	T		T

semantic entailment not valid

Therefore, by the Soundness Th.,

$p \to q \vdash q$ is not provable!

p	q	$p \to q$, $\neg q$, p	$\vdash$	$\bot$
F	F	T	F	F
F	T	T	F	F
T	F	F	T	F
T	T	T	F	F

Valid semantic entailment!

Therefore, $p \to q, \neg q, p \vdash \bot$ is provable by the Completeness Th.

p	q	$p \to (q \to p)$	
F	F	T	T
F	T	T	F
T	F	T	T
T	T	T	T

$\neg p, \neg q \vdash p \to (q \to p)$

$\neg p, q \vdash p \to (q \to p)$

$p, \neg q \vdash p \to (q \to p)$

$p, q \vdash p \to (q \to p)$