1.5.2. $p \to (q \vee r)$

(a) $q \vee (\neg p \vee r)$

(b) $q \wedge \neg r \to p$

$v(q) = T \quad v(r) = F \quad v(p) = F$

$v(p \to (q \vee r)) = T$

$v(q \wedge \neg r \to p) = F$

} So, not semantically equivalent.

$\{\neg, \wedge, \vee, \rightarrow\}$

$\phi \wedge \psi \equiv \neg(\neg\phi \vee \neg\psi)$

$\phi \rightarrow \psi \equiv \neg\phi \vee \psi$

Therefore, $\{\neg, \vee\}$ is adequate.

1.5.3.

(a) $\{\neg, \wedge\}$

$$\phi \vee \psi \equiv \neg(\neg\phi \wedge \neg\psi)$$

Therefore, $\{\neg, \wedge\}$ is adequate because $\{\neg, \vee\}$ is.

$\{\neg, \rightarrow\}$: exercise

$\{\rightarrow, \bot\}$

$$\neg\phi \equiv \phi \rightarrow \bot$$

$$\phi \vee \psi \equiv (\phi \rightarrow \bot) \rightarrow \psi$$

$p \vee r \vee \neg q \vee s \vee \neg t$

$v(p) = F$

$v(r) = F$

$v(q) = T$

$v(s) = F$

$v(t) = T$

p	q	$(p \to q) \wedge (q \to p)$		
F	F	T	T	T
→ F	T	T	F	F
→ T	F	F	F	T
T	T	T	T	T

$$(p \vee \neg q) \wedge (\neg p \vee q)$$

Equivalent formula in CNF

p	q	r	$(p \vee \neg q) \wedge (q \vee r)$
F	F	F	F
F	F	T	T
F	T	F	F
F	T	T	F
T	F	F	F
T	F	T	T
T	T	F	T
T	T	T	T

$(\neg p \wedge q) \rightarrow (p \wedge (r \rightarrow q))$

$\Big\downarrow$ IMPL_FREE

$\neg(\neg p \wedge q) \vee (p \wedge (\neg r \vee q))$

$\Big\downarrow$ NNF

$(p \vee \neg q) \vee (p \wedge (\neg r \vee q))$

$\Big\downarrow$ CNF'

$\text{DISTR}(\text{CNF}'(p \vee \neg q), \text{CNF}'(p \wedge (\neg r \vee q)))$

$\text{DISTR}(p \vee \neg q, p \wedge (\neg r \vee q))$

$\text{DISTR}(p \vee \neg q, p) \wedge \text{DISTR}(p \vee \neg q, \neg r \vee q)$

$(p \vee \neg q \vee p) \wedge (p \vee \neg q \vee \neg r \vee q)$ $\rightsquigarrow$ $p \vee \neg q$

simplification

1.5.15.

$$p_1 \wedge p_2 \wedge \dots \wedge p_{k_i} \rightarrow q_i \quad \equiv$$

$$\neg (p_1 \wedge p_2 \wedge \dots \wedge p_{k_i}) \vee q_i \quad \equiv$$

$$\neg p_1 \vee \neg p_2 \vee \dots \vee \neg p_{k_i} \vee q_i$$

1.5.15. (d)

$$(p \wedge q \wedge s \to \bot) \wedge (q \wedge r \to p) \wedge (\top \to s)$$

p	q	r	s	
			✓	
F	F	F	T	← satisfying assignment