2.4.2.

$$\phi \stackrel{\text{def}}{=} \forall x \, \exists y \, \exists z \left(P(x,y) \wedge P(z,y) \wedge \left(P(x,z) \rightarrow P(z,x) \right) \right)$$

(a) $A \stackrel{\text{def}}{=} \mathbb{N}$

$$P^{\mathcal{M}} \stackrel{\text{def}}{=} \{ (m,n) \mid m < n \}$$

$$m \; P^{\mathcal{M}} \; n \stackrel{\text{def}}{\iff} m < n$$

Abusing notation:

$$\forall x \in \mathbb{N} \; \exists y \in \mathbb{N} \; \exists z \in \mathbb{N}$$

$$\left(x < y \wedge z < y \wedge \left(x < z \rightarrow z < x \right) \right)$$

$$\mathcal{M} \models \phi$$

2.4.3.

$$\forall x \neg p(x,x)$$

$$A = \{ \text{apple} \}$$

$$p^{\mathcal{M}_1} = \{\} \qquad p^{\mathcal{M}_2} = \{(\text{apple}, \text{apple})\}$$

$$\mathcal{M}_1 \models \forall x \neg p(x,x) \qquad \mathcal{M}_2 \not\models \forall x \neg p(x,x)$$

2.4.12.

(a) $\forall x \forall y (S(x,y) \to S(y,x)) \to \forall x \neg S(x,x) = \phi$

$A = \{\text{apple}\}$

$S^{M_2} = \{(\text{apple}, \text{apple})\}$

$M_2 \not\models \phi$

$A = \{\text{apple}\}$

$S^{M_1} = \{\}$

$M_1 \models \phi$

$A = \mathbb{N}$

$n \, S^{M_3} \, m \iff n = m$

$M_3 \not\models \phi$